AIP Conference Proceedings

A discussion of “On Bayesian estimation of a survival curve: Comparative study and examples”

A. Polpo, D. Sinha, P. R. Simioni, and C. A. B. Pereira

Citation: AIP Conf. Proc. **1490**, 268 (2012); doi: 10.1063/1.4759611
View online: http://dx.doi.org/10.1063/1.4759611
View Table of Contents: http://proceedings.aip.org/dbt/dbt.jsp?KEY=APCPCS&Volume=1490&Issue=1
Published by the American Institute of Physics.

A Discussion of “On Bayesian Estimation of a Survival Curve: Comparative Study and Examples”

Polpo, A.*, Sinha, D.†, Simioni, P. R.* and Pereira, C. A. B.**

*Federal University of São Carlos, Brazil
†Florida State University, USA
**University of São Paulo, Brazil

Abstract. A small mistake when introducing a new methodology could completely ruin the possibility a success of it. In the context of reliability for the case of series systems, Polpo and Sinha [8] presented the corrected version of an estimator introduced by Salinas-Torres et al. [10]. Recently, Salinas et al. [9] applied the former results to built estimators for survival functions and compare them to standard ones. Again, the same kind of equivocal development was committed. Besides introducing the correct estimates, we show the desired asymptotic properties in fact hold.

Keywords: Competing-risks, right censoring, series system, Bayesian nonparametric.
PACS: 02.50.Tt, 02.50.Ng, 02.60.Gf.

1. INTRODUCTION

In survival analysis, data censored at right has been studied by many authors. For a review, we refer to Ibrahim et al. [3], under a Bayesian perspective, and Lawless [5] for a frequentist view. A review of Nonparametric Bayesian data analysis can be found in Müller and Quintana [6]. The present paper discusses the estimator introduced by Polpo and Sinha [8] performing an analysis similar to the one of Salinas et al. [9]. In addition we discuss the question of choosing an appropriate prior for the non-parametric estimator of Polpo and Sinha [8].

All notations used here are similar to those used by Salinas-Torres et al. [10]. Consider a series system with r components. Let X_j, $j = 1,\ldots,r$ the failure time of the j-th component. The observed sample of size n is (Z_i, δ_i), $i = 1,\ldots,n$, where $Z_i = \min(X_{1i},\ldots,X_{ri})$, and $\delta = j$ if the j-th component was the responsible to system failure, that is $Z_i = X_{ji}$, $j = 1,\ldots,r$. Also, consider that the failure time of all components are independent.

Define $S_j^*(t) = \Pr(Z > t, \delta = j)$ and $S_j(t) = \Pr(Z > t)$ as the subsurvial and survival functions of the j-th component, respectively. The objective is to construct a nonparametric estimator of $S_j(\cdot)$. Restricting ourselves to the case with two components ($r = 2$), Peterson [7] define a functional that relates the subsurvival functions with the survival function of a component.

XI Brazilian Meeting on Bayesian Statistics
AIP Conf. Proc. 1490, 268-277 (2012); doi: 10.1063/1.4759611

Theorem 1 (Theorem 2.1 of Peterson [7])

1. *The subsurvival functions* $S_1^*(\cdot)$ *and* $S_2^*(\cdot)$ *determine (uniquely) the survival function* $S_1(t)$ *for* $t \leq t^*$ *according to the following explicit expression:*

$$S_1(t) = \varphi(S_1^*(\cdot), S_2^*(\cdot), t), \tag{1}$$

where

$$\varphi(S_1^*(\cdot), S_2^*(\cdot), t) = \exp\left[\oint_0^t \frac{\mathrm{d}S_1^*(s)}{S_1^*(s) + S_2^*(s)}\right] \times \exp\left[\sum_{\substack{\text{s: jump point of } S_1^*(\cdot)\\ t \leq t^*}} \log\left(\frac{S_1^*(s^+) + S_2^*(s^+)}{S_1^*(s^-) + S_2^*(s^-)}\right)\right],$$

and $\oint_0^t$ *means integration over the (open) intervals of points less than t for which* $S_1^*(\cdot)$ *is continuous.*

2. *If* $S_1(t^*) = 0$*, then* $S_1(t) = 0$ *for* $t > t^*$*.*

Proof. See Peterson [7]. □

Using (1), Peterson [7] express the Kaplan-Meier estimator in terms of empirical estimator of subsurvival function, $S_{jn}^*(t) = \frac{1}{n}\sum_{i=1}^n \mathbb{I}(Z_i > t, \delta_i = j)$, $j = 1, 2$, where $\mathbb{I}(A)$ is the indicator function of set A. Similarly, Salinas-Torres et al. [10], using the Dirichlet process [2, 11], developed a nonparametric Bayesian estimator for subsurvival functions, and for component's survival function, by functional (1). Polpo and Sinha [8] presented a correction of these estimates. Here it is shown that these estimates are in fact consistent and it performs better as the sample size increases as expected. On the other hand, the version presented in [10] is inconsistent: their nonparametric estimates became worse as the sample size increases.

This paper is organized as follows. Section 2 describes the correction made in the original estimator. The description and the use of the prior are discussed in Section 3. Using simulated data, Section 4 illustrates a comparison between the Bayesian nonparametric estimate and the celebrated Kaplan-Meyer estimate [4]. Final remarks are the subjects of Section 5.

2. CORRECTION OF BAYESIAN NONPARAMETRIC ESTIMATOR

Theorem 1 of Salinas-Torres et al. [10] is the main result of their paper, and Polpo and Sinha [8] presented the correct version of this result. In fact the result in the old version is responsible for the negative results of the Bayesian estimator compared with the Kaplan-Meyer.

The following steps introduce the main results to understand the estimator, highlight the place where the problem happens, and present the proper correction.

Returning to the problem of more than two components ($r > 1$), let Δ the subset of indexes $\{1,\ldots,r\}$ and Δ^c its complement. Considering $\mathscr{X} = (0,\infty)$, $\mathscr{A} = \mathscr{B}_{(0,\infty)}$ (Borel σ-algebra), $\boldsymbol{S}^*(\cdot) = (S_1^*(\cdot),\ldots,S_r^*(\cdot))$, the prior for $\boldsymbol{S}^*$ is defined by

$$\boldsymbol{S}^*(t) \sim \mathscr{D}(a_1(t,\infty),\ldots,a_r(t,\infty)), \tag{2}$$

for which a_j ($j = 1,\ldots,r$) are finite positive measures on $(\mathscr{X},\mathscr{A})$ and $\mathscr{D}$ is a Dirichlet distribution. Then, the induced prior for S_Δ^*, defined by $\Pr(Z > t, \delta \in \Delta)$, is

$$S_\Delta^*(t) \sim \mathrm{Beta}[c_r S_{\Delta,0}^*(t), c_r(1 - S_{\Delta,0}^*(t))], \tag{3}$$

where $c_r = \sum_{j=1}^r a_j(0,\infty)$ and the prior mean is $S_{\Delta,0}^*(t) = \sum_{j\in\Delta} a_j(t,\infty)/c_r$. The posterior distribution of $\boldsymbol{S}^*(t)$ is

$$\boldsymbol{S}^*(t) \mid Data \sim \mathscr{D}\left[a_1(t,\infty) + nS_{1n}^*(t),\ldots,a_r(t,\infty) + nS_{rn}^*(t)\right].$$

The Bayesian nonparametric estimators are then

$$\widehat{S}_\Delta^*(t) = p_n S_{\Delta,0}^*(t) + (1 - p_n) S_{\Delta,n}^*(t), \tag{4}$$

where $p_n = c_r/(c_r + n)$, and

$$\widehat{S}(t) = \widehat{S}_\Delta^*(t) + \widehat{S}_{\Delta^c}^*(t). \tag{5}$$

In the sequel we present the main result of Salinas-Torres et al. [10] and pin-point the place where need a correction.

Let $Z_{(1)} < \cdots < Z_{(m)}$, $m(\le n)$ distinct order statistics. As defined by Salinas-Torres et al. [10],

$$i_\Delta(t) = \exp\left\{ \frac{-1}{c_r + n} \sum_{j\in\Delta^c} \oint_0^t \frac{\mathrm{d}a(s,\infty)}{\widehat{S}(s)} \right\},$$

$$\pi_\Delta(t) = \prod_{i:Z_{(i)}\le t} \frac{\sum_{j=1}^r a_j(Z_{(i)},\infty) + n_i - d_i}{\sum_{j=1}^r a_j(Z_{(i)},\infty) + n_i},$$

$n_i = \sum_{k=1}^n \mathbb{I}(Z_k \ge Z_{(i)})$, and $d_i = \sum_{k=1}^n \mathbb{I}(Z_k = Z_{(i)}, \delta_k \in \Delta)$.

Theorem 2 (Theorem 1 of Salinas-Torres et al. [10]) *Suppose that the function $f(s) = (a_1(s,\infty),\ldots,a_r(s,\infty))$ is continuous on $(0,t)$, for each $t > 0$, and S_Δ and S_{Δ^c} have no common discontinuities then, for $t \le Z_{(m)}$,*

$$\widehat{S}_\Delta(t) = \varphi(\widehat{S}_\Delta^*, \widehat{S}_{\Delta^c}^*; t) = \widehat{S}(t) i_\Delta(t) \pi_\Delta(t),$$

is the Bayes estimator of $S_\Delta(t)$ under the quadratic loss function.

To prove Theorem 2, the authors used the fact that

$$\mathrm{d}\widehat{S}^*_\Delta(s) = \mathrm{d}\widehat{S}(s) - \frac{\sum\limits_{j\in\Delta^c} \mathrm{d}a(s,\infty)}{(c_r+n)}$$

to obtain the following steps:

$$\begin{aligned}
\exp\left\{\oint_0^t \frac{\mathrm{d}\widehat{S}^*_\Delta(s)}{\widehat{S}(s)}\right\} &= \exp\left\{\oint_0^t \frac{\mathrm{d}\widehat{S}(s) - \left(\sum\limits_{j\in\Delta^c} \mathrm{d}a(s,\infty)\right)/(c_r+n)}{\widehat{S}(s)}\right\} \qquad (6)\\
&= \exp\left\{\oint_0^t \frac{\mathrm{d}\widehat{S}(s)}{\widehat{S}(s)}\right\} \exp\left\{\frac{-1}{c_r+n}\oint_0^t \frac{\sum\limits_{j\in\Delta^c} \mathrm{d}a(s,\infty)}{\widehat{S}(s)}\right\}\\
&= \widehat{S}(t)\, i_\Delta(t).
\end{aligned}$$

Note that to obtain the last line they used that $\widehat{S}(t) = \exp\left\{\oint_0^t \frac{\mathrm{d}\widehat{S}(s)}{\widehat{S}(s)}\right\}$. However, this is only true when $\widehat{S}(t)$ is absolute continuous [see 1]. However, the Bayesian nonparametric estimator of $\widehat{S}(t)$ under quadratic loss function is not absolute continuous. Also, the integration in the left side of (6) is taken only over the intervals where $\widehat{S}^*_\Delta$ is continuous (see Theorem 1). These steps produced a wrong result. The Theorem 2 was corrected by Polpo and Sinha [8] via redefining $i_\Delta(t)$ as

$$\tilde{i}_\Delta(t) = \exp\left\{\frac{1}{c_r+n}\sum_{j\in\Delta}\oint_0^t \frac{\mathrm{d}a(s,\infty)}{\widehat{S}(s)}\right\}, \qquad (7)$$

and rewritten as:

Theorem 3 *Suppose that the function* $f(s) = (a_1(s,\infty),\ldots,a_r(s,\infty))$ *is continuous on* $(0,t)$*, for each* $t>0$*, and* S_Δ *and* S_{Δ^c} *have no common discontinuities then, for* $t \le Z_{(m)}$*,*

$$\widetilde{S}_\Delta(t) = \varphi(\widehat{S}^*_\Delta, \widehat{S}^*_{\Delta^c}; t) = \tilde{i}_\Delta(t)\pi_\Delta(t),$$

is the Bayes estimator of $S_\Delta(t)$ *under the quadratic loss function.*

Proof. Substituting $\mathrm{d}\widehat{S}^*_\Delta(t)$ in the proof of Theorem 2 by $\mathrm{d}\widehat{S}^*_\Delta(t) = \left(\sum_{j\in\Delta}\mathrm{d}a(t,\infty)\right)/(c_r+n)$ the first term in (2.4) of Salinas-Torres et al. [10] becomes $\tilde{i}_\Delta(t)$ defined in (7). $\mathrm{d}\widehat{S}^*_\Delta(t) = \left(\sum_{j\in\Delta}\mathrm{d}a(t,\infty)\right)/(c_r+n)$ is a consequence of

$$\begin{aligned}
\widehat{S}^*_\Delta(t) &= p_n S^*_{\Delta,0}(t) + (1-p_n) S^*_{\Delta,n}(t)\\
&= \frac{\sum\limits_{j\in\Delta} a(t,\infty)}{c_r+n} + \frac{\sum\limits_{i=1}^{n} I(Z_i>t,\delta\in\Delta)}{c_r+n},
\end{aligned}$$

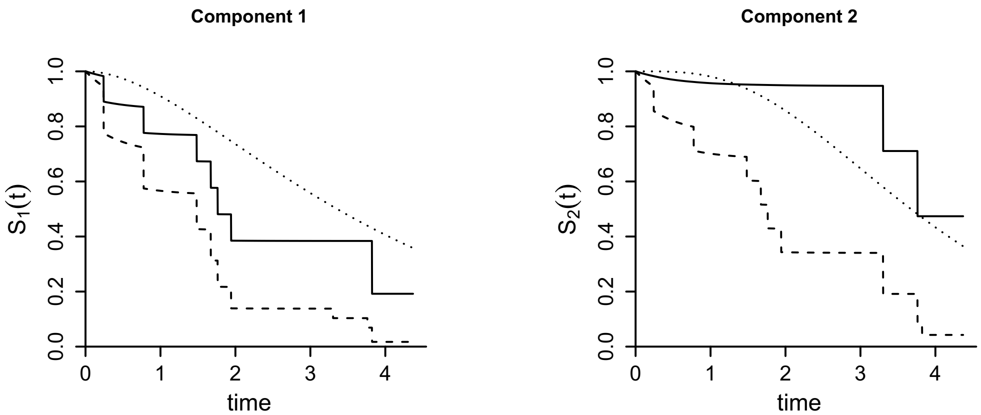

FIGURE 1. Nonparametric estimator of component's survival for sample size 10 units: (a) Component 1; (b) Component 2. Dot line – true survival; solid line – corrected estimator; and dashed line – original estimator.

and the function $\sum_{i=1}^{n} I(Z_i > t, \delta \in \Delta)$ is constant, except at jump points, so its derivative is zero in continuous intervals. To end the proof simply follow the steps of Salinas-Torres et al. [10]. □

To investigate the differences between the estimators of Salinas-Torres et al. [10] and of Polpo and Sinha [8], we use the following simulation study: consider a series system with two components, X_1 and X_2. The observed data is $T_i = \min(X_{1i}, X_{2i})$ and $\delta_i = 1$ if X_1 was the first component to fail, or 2 otherwise. The model used to simulate the data considers X_1 as gamma distributed, mean 4 and variance 8, and X_2 also as gamma distributed, mean 4 and variance 4. We simulate observations of this series system using three samples: $n = 10$, $n = 100$ and $n = 1000$. The results are presented in Figure 1, 2 and 3. As expected, the Polpo and Sinha [8] estimator is consistent, approaches to the "true" distribution with the increase of the sample size. On the other hand, the estimator of Salinas-Torres et al. [10] have a bad performance, do not approach the distribution that simulates the data.

3. PRIOR SPECIFICATION

An important aspect of the Bayesian estimator is the possibility to use prior information that is elicited by the prior distribution. In the nonparametric estimation, however, the choice of the finite measure that defines the Dirichlet Process is a difficult task whenever one wants to have the prior well described. Note that these finite measures, a_j, are defined for the vector of sub-survival functions, then $a_j(t;\infty)$ can be viewed as prior measures for the sub-survival functions $S_j^*(t)$.

Property 1 of Peterson [7] states the inverse relation of (1) by $S_1^*(t) = \int_t^{\infty} S_2(s)[-\mathrm{d}S_1(s)]$. Using this property, one can elicit a prior for the survival functions and then evaluate for the sub-survival. For example, considering $r = 2$, and as prior

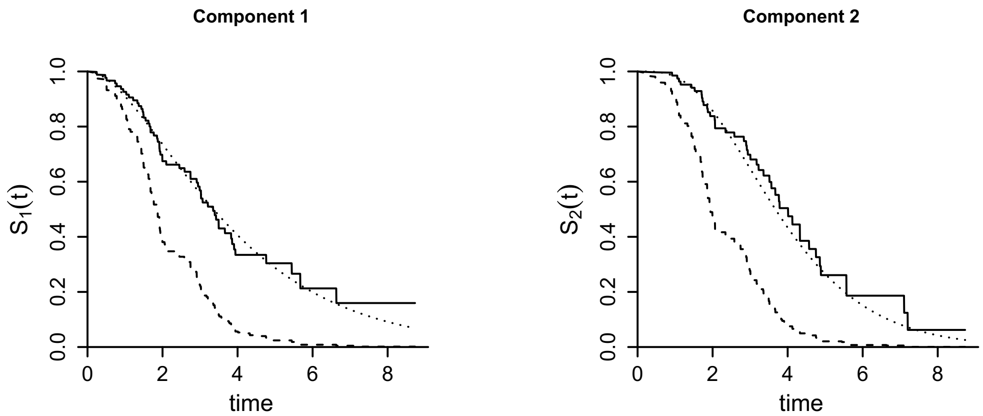

FIGURE 2. Nonparametric estimator of component's survival for sample size 100 units: (a) Component 1; (b) Component 2. Dot line – true survival; solid line – corrected estimator; and dashed line – original estimator.

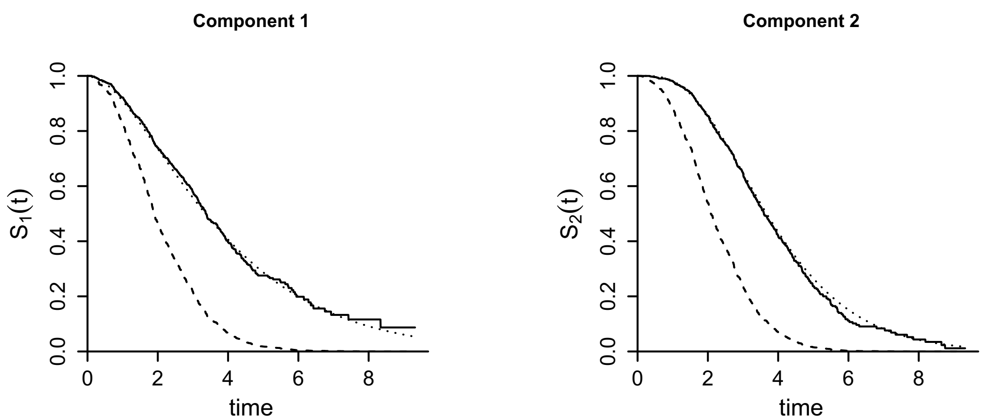

FIGURE 3. Nonparametric estimator of component's survival for sample size 1000 units: (a) Component 1; (b) Component 2. Dot line – true survival; solid line – corrected estimator; and dashed line – original estimator.

for $S_1(t)$ and $S_2(t)$ independent exponential distributions with mean m. Then, applying Property 1 of Peterson [7] we get

$$a_j(t,\infty) = \frac{\exp\{-2t/m\}}{2}, j = 1,2.$$

The formal measure of the prior space is $c_r = \sum_{j=1}^{r} a_j(0,\infty)$. From (4) and (5), c_r can be viewed as the importance of the prior in the estimator, called here as prior weight. For instance, if $c_r = 1$ the prior has the same importance as one sample unit. For example, if a company "knows" the distribution of each component (by some individual test or project specification) and they are testing the components working together in the series system, probably they will give more importance to the prior than someone without any information. In fact they would accept drastic changes from prior to posterior only if the data brings contusing or huge amount information compared with the prior one.

In the example above, we can redefine the prior by

$$a_j(t,\infty) = \frac{k\exp\{-2t/m\}}{2}, j = 1,2.$$

In this case $c_r = a_1(0,\infty) + a_2(0,\infty) = k$; we can specify the prior weight as desirable. This exponential prior measure is a good option for prior elicitation, where the "researcher" only need to specify properly the prior mean and the weight.

4. COMPARATIVE STUDY

Salinas et al. [9] did a comparative study of the Bayesian nonparametric estimator using the estimator given by Salinas-Torres et al. [10]. In this section we redo the comparative studies of Salinas et al. [9] with appropriate changes in the prior measures and the Polpo and Sinha [8] estimator. To evaluate the Bayesian nonparametric estimator we use the same numerical method described in Salinas et al. [9]. Also, we use the same statistics L_2–norm and MSVP, a cross-validation measure, to compare the estimator with the alternative Kaplan-Meier estimator.

Following Salinas et al. [9], we draw random samples of size n of a system with two components. Our interest consist in the estimation of component X_1. The component X_2 is considered as a random censoring variable. We specify the distribution of X_1 as exponential distribution with mean 3, and X_2 as gamma distributions with mean μ and variance μ. We considered three samples size $n = \{50,100,200\}$, and three values for $\mu = 1,2,4$ which correspond 75, 55 and 30 percent of censoring, respectively. Then we have 9 different scenarios in our simulation study. For each scenario we simulated 1,000 different samples.

Salinas et al. [9] suggested the prior measure as $a_j(t,\infty) = M_j - t$, $0 < t < M_j$, $j = 1,2$. They considered $M_1 = M_2$, and different values in of M_j in each scenario. Then, the considered prior weight is $c_r = M_1 + M_2$. We found that the smallest prior weight suggested in their work was 23% when compared with sample size (this happen in the case of $n = 200$ with 30% of censoring). We understand that for a "non-informative" prior, Salinas et al. [9] used a large weight in consideration of their prior. We changed the prior to have a smaller weight suggesting the use of $a_j(t,\infty) = (M_j - t)/M_j$, $0 < t < M_j$, $j = 1,2$, which implies that $c_r = 2$ for any value of M_j. We consider for all scenarios $M_1 = M_2 = 60$, and the biggest prior weight is around 4%.

L_2–norm measure is defined as

$$||S_1 - A||_2 = \left\{ \int_0^{Z^*_{(m)}} (S_1(t) - A(t))\mathrm{d}t \right\},$$

where $S_1(t)$ is the "true" survival (exponential distribution), and $A(t)$ is one of the three possible estimators: $\widetilde{S}_1(t)$ for the Polpo and Sinha [8] Bayesian nonparametric estimator; $\mathrm{KM}(t)$ for the Kaplan-Meier estimator; and $\widehat{S}_1(t)$ for the original Bayesian estimator of Salinas-Torres et al. [10]. Tables 1, 2 and 3 presents the descriptive statistics (minimum, maximum, 25th percentile, 75th percentile, and standard deviation) of the 1,000 different samples. The results shows that the Kaplan-Meier (KM) and the Bayesian nonparametric estimator ($\widetilde{S}_1$) have similar performance, and the original Bayesian nonparametric estimator ($\widehat{S}_1$) does not fit the data.

TABLE 1. Descriptive statistics of L_2–norm, $n = 50$.

	Mean	Median	Min	Max	P25	P75	SD
75% of censoring							
$\|\|S_1 - \widetilde{S}_1\|\|_2$	0.215	0.174	0.035	0.976	0.113	0.262	0.144
$\|\|S_1 - \mathrm{KM}\|\|_2$	0.207	0.172	0.042	0.950	0.126	0.251	0.121
$\|\|S_1 - \widehat{S}_1\|\|_2$	0.741	0.742	0.495	0.907	0.699	0.783	0.063
55% of censoring							
$\|\|S_1 - \widetilde{S}_1\|\|_2$	0.215	0.176	0.043	1.050	0.122	0.266	0.131
$\|\|S_1 - \mathrm{KM}\|\|_2$	0.196	0.169	0.051	0.955	0.130	0.230	0.103
$\|\|S_1 - \widehat{S}_1\|\|_2$	0.634	0.633	0.443	0.801	0.590	0.677	0.062
30% of censoring							
$\|\|S_1 - \widetilde{S}_1\|\|_2$	0.195	0.168	0.050	0.738	0.121	0.242	0.102
$\|\|S_1 - \mathrm{KM}\|\|_2$	0.174	0.158	0.053	0.621	0.121	0.209	0.077
$\|\|S_1 - \widehat{S}_1\|\|_2$	0.518	0.520	0.251	0.734	0.466	0.571	0.077

TABLE 2. Descriptive statistics of L_2–norm, $n = 100$.

	Mean	Median	Min	Max	P25	P75	SD
75% of censoring							
$\|\|S_1 - \widetilde{S}_1\|\|_2$	0.196	0.157	0.043	1.098	0.108	0.251	0.127
$\|\|S_1 - \mathrm{KM}\|\|_2$	0.188	0.164	0.041	1.071	0.121	0.223	0.106
$\|\|S_1 - \widehat{S}_1\|\|_2$	0.796	0.796	0.642	0.940	0.767	0.827	0.046
55% of censoring							
$\|\|S_1 - \widetilde{S}_1\|\|_2$	0.176	0.150	0.042	0.834	0.102	0.220	0.100
$\|\|S_1 - \mathrm{KM}\|\|_2$	0.162	0.145	0.039	0.796	0.109	0.192	0.080
$\|\|S_1 - \widehat{S}_1\|\|_2$	0.670	0.669	0.524	0.825	0.640	0.700	0.044
30% of censoring							
$\|\|S_1 - \widetilde{S}_1\|\|_2$	0.152	0.132	0.047	0.522	0.097	0.192	0.073
$\|\|S_1 - \mathrm{KM}\|\|_2$	0.136	0.125	0.038	0.496	0.096	0.165	0.056
$\|\|S_1 - \widehat{S}_1\|\|_2$	0.543	0.543	0.380	0.695	0.505	0.581	0.054

Table 4 presents the mean of MSPV statistic, defined as

$$MSVP = \frac{1}{n}\sum_{i=1}^{n}\left(A(Z_i) - A_{(-i)}(Z_i)\right)^2,$$

where A is one of the two estimators ($\widetilde{S}$ and KM), and $A_{(-i)}$ is the estimates of survival function from a sample without the i-th observation. Again, the results show that the Bayesian nonparametric estimator and the KM are very similar. The performance of KM was a slightly better than the Bayesian nonparametric estimator. Probably this is a consequence of a bad choice of prior measures (as discussed before). Note that, despite the prior being a uniform distribution, in this case it is not non-informative prior.

TABLE 3. Descriptive statistics of L_2–norm, $n = 200$.

	Mean	Median	Min	Max	P25	P75	SD
75% of censoring							
$\|\|S_1 - \widetilde{S}_1\|\|_2$	0.172	0.137	0.039	0.807	0.095	0.224	0.110
$\|\|S_1 - \text{KM}\|\|_2$	0.164	0.143	0.049	0.786	0.104	0.197	0.090
$\|\|S_1 - \widehat{S}_1\|\|_2$	0.827	0.830	0.703	0.929	0.806	0.850	0.032
55% of censoring							
$\|\|S_1 - \widetilde{S}_1\|\|_2$	0.146	0.124	0.036	0.603	0.087	0.184	0.083
$\|\|S_1 - \text{KM}\|\|_2$	0.134	0.118	0.040	0.535	0.091	0.161	0.066
$\|\|S_1 - \widehat{S}_1\|\|_2$	0.693	0.693	0.575	0.784	0.673	0.715	0.032
30% of censoring							
$\|\|S_1 - \widetilde{S}_1\|\|_2$	0.112	0.099	0.037	0.368	0.072	0.140	0.053
$\|\|S_1 - \text{KM}\|\|_2$	0.102	0.094	0.035	0.344	0.073	0.122	0.041
$\|\|S_1 - \widehat{S}_1\|\|_2$	0.556	0.556	0.441	0.664	0.530	0.582	0.039

TABLE 4. Mean of MSPV for ($\widetilde{S}_1$) and [KM] estimates.

Percent of censoring	$n = 50$	$n = 100$	$n = 200$
75	(0.0052) [0.0036]	(0.0019) [0.0013]	(0.0007) [0.0004]
55	(0.0030) [0.0020]	(0.0010) [0.0007]	(0.0003) [0.0002]
30	(0.0011) [0.0007]	(0.0003) [0.0002]	(0.0001) [0.0001]

5. FINAL REMARKS

Along the above sections we have revisited the Bayesian nonparametric estimation of survival functions for series system or competing-risks models showing the proper use of it. Also this paper discusses the choice of a prior measure, the parameter measure, of the the Dirichlet Process for the Bayesian non-parametric estimator of Polpo and Sinha [8], presents simulation studies similar to those of Salinas et al. [9] and compares the Bayesian non-parametric estimators with the Kaplan-Meier estimator. The results presented show that both estimators performed very similar for point estimation. However it remains the need of addressing the question of computing credible (confidence) intervals: this is left for a future work.

With the use of the correct estimates presented in Polpo and Sinha [8] we have shown the consistence of the estimators. Now we have a good continuous estimator of a continuous function, as desirable. Note that although Kaplan-Meyer has a similar performance, respecting to consistency, it is in fact a step function.

REFERENCES

1. P.K. Andersen, B. Ørnulf, R.D. Gill, and N. Keiding. *Statistical Models Based on Countig Processes*. Springer series in statistics. Spinger-Verlag, New York, 1992.
2. T.S. Ferguson. A bayesian analysis of some nonparametric problems. *The Annals of Statistics*, 1: 209–230, 1973.
3. J. G. Ibrahim, M-H. Chen, and D. Sinha. *Bayesian Survival Analysis*. Springer, June 2001.

4. E.L. Kaplan and P. Meier. Nonparametric estimation from incomplete observations. *Journal of the American Statistical Association*, 53:457–481, 1958.
5. J. F. Lawless. *Statistical models and methods for lifetime data*. Wiley-Interscience, 2nd edition, 2003.
6. P. Müller and F. Quintana. Nonparametric bayesian data analysis. *Statistical Science*, 19(1):95–110, 2004. doi: 10.1214/088342304000000017.
7. A.V. Peterson. Expressing the kaplan-meier estimator as a function of empirical subsurvival functions. *Journal of the American Statistical Association*, 72(360):854–858, 1977.
8. A. Polpo and D. Sinha. Correction in bayesian nonparametric estimation in a series system or a competing-risks model. *Statistics & Probability Letters*, 81(12):1756–1759, December 2011. doi: 10.1016/j.spl.2011.07.023.
9. V. H. Salinas, J. S. Romeo, and A. Peña. On bayesian estimation of a survival curve: comparative study and examples. *Computational Statistics*, 25:375–389, 2010.
10. V. Salinas-Torres, C.A.B. Pereira, and R. Tiwari. Bayesian nonparametric estimation in a series system or a competing risks model. *Journal of Nonparametric Statistics*, 14:449–458, 2002.
11. V.H. Salinas-Torres, C.A.B. Pereira, and R.C. Tiwari. Convergence of dirichlet measures arising in context of bayesian analysis of competing risks models. *Journal of Multivariate Analysis*, 62:24–35, 1997.